

ACAA

LANGMUIR TURBULENCE IN SPACE PLASMAS

Crockett L. Grabbe and James P. Sheerin

Department of Physics and Astronomy -- University of Iowa
Iowa City, Iowa 52242
UNITED STATES OF AMERICA

NAGW-364
7N-15

48015

CR

P-28

ABSTRACT

The basic developments in Langmuir turbulence in recent years are reviewed. The increasing importance of this subject to space plasma studies is emphasized with a discussion of observations upstream of planetary bow shocks, and its role in type III solar radio bursts and ionospheric modification.

1. INTRODUCTION

The problem of the nonlinear development of Langmuir waves has been an important area of research for over twenty years [Kadomtsev, 1964; Tsytovich, 1968; Tsytovich, 1977]. The classic problem associated with the theory of the development was that of the tendency of the energy of the k -spectrum to cascade toward lower k 's (higher wavelengths). Since the Landau and collisional damping decrease as k decreases, it appeared as if the spectrum of waves would condense towards $k=0$. The mechanism for the resolution or dissipation of this energy was not known. Then Zakharov [1972] showed that the mechanism for the resolution of this tendency of the spectrum to pile up at $k=0$ could be identified as a process known as collapse of the Langmuir wavepacket.

Zakharov derived two nonlinearly coupled equations for high-frequency (Langmuir) and low frequency (ion acoustic) wave turbulence. These two equations have been the focus of most of the research on Langmuir turbulence in the last thirteen years. Along with the theoretical developments there has been increased interest in Langmuir turbulence in the space and astrophysical plasma community because of the observation of this turbulence in the solar wind upstream of the planetary bow shock and because of its apparent role in the generation of type III solar radio bursts and in ionospheric modification.

In this paper a review of some of the aspects of Langmuir turbulence described by the Zakharov equations will be made, then the rest of the paper devoted to some specific applications in space and astrophysical plasmas where it may be important. For a more extensive review of the basic principles of "strong" Langmuir turbulence see the works by Tsytovich [1977], Thornhill and ter Haar [1978], Rudakov and Tsytovich [1978], and Goldman [1984]. Also, a guide to many of the major literature sources on strong and weak turbulence is contained in Grabbe [1984].

In Sec. 2 the weak turbulence theory of Langmuir waves is briefly reviewed. In Sec. 3 the Zakharov equations are reviewed. Then in Sec. 4 the parametric decay and oscillating two-stream (modulational) instabilities are studied. In Sec. 5 we examine the one-dimensional case and the soliton solutions that occur in this case. In Sec. 6 the multi-dimensional self-similar solutions corresponding to Langmuir collapse and related physics are examined. In Sec. 7 we apply the ideas on parametric instabilities and collapse to ionospheric modification. In Sec. 8 we examine Langmuir turbulence observed upstream of planetary bow shocks. In Sec. 9 the theory of the generation of type III solar radio burst is reviewed. A summary and conclusion is contained in Sec. 10.

2. WEAK TURBULENCE PROCESSES

Weak turbulence involves a combination of the quasilinear approximation for wave-particle interactions and the random-phase approximation for wave-wave interactions. An essential part of these approximations is that the linear wave concepts still hold and are valid, and that nonlinear modifications are slow compared to the inverse of characteristic linear frequencies and small compared to linear wave amplitudes. They are not valid if the wave interactions are very coherent. As we will see these concepts will normally break down in Langmuir turbulence, but it is instructive to examine these processes.

There are four interactions that can occur in weak turbulence theory. Letting ℓ denote Langmuir waves, s ion acoustic waves, t transverse (ordinary mode) waves, i ions, and e -electrons these processes are:

$$(1) \text{ Parametric decay: } \ell_1 \leftrightarrow \ell_2 + s$$

$$(2) \text{ Scattering off ions and electrons: } \ell_1 + i_1 \leftrightarrow \ell_2 + i_2$$

$$\ell_1 + e_1 \leftrightarrow \ell_2 + e_2$$

$$(3) \text{ Four-wave scattering: } \ell_1 + \ell_2 \leftrightarrow \ell_3 + \ell_4$$

$$(4) \text{ Conversion into transverse waves: } \ell_1 + \ell_2 \leftrightarrow t$$

These are the wave-wave and wave-particle interactions that are allowed in weak turbulence theory. They are subject to conditions for energy and momentum conservation. For the parametric decay, for example, these are the familiar Manley-Rowe relations:

$$\omega_1 = \omega_2 + \omega_s \quad (1)$$

$$\vec{k}_1 = \vec{k}_2 + \vec{k}_s$$

Parametric decay is the dominant process in weak turbulence, and it causes the Langmuir waves to cascade to lower k . However, decay is only consistent with the Langmuir and acoustic wave dispersion relations and the Manley-Rowe relations provided

$$k > \sqrt{\mu} k_D / 3 \quad (2)$$

where k_D is 2π divided by the Debye length and μ is the mass ratio. For k 's below this critical value the four-wave scattering processes are normally dominant. This process leads to a thermal equilibrium of plasma waves. It dominates over the wave-particle scattering, which tends to cause a thermalization of the waves with the ions, and Landau and collisional damping, which are small at these low k -values. Thus we have the paradox that if the waves are pumped at higher k -values, they pile up and thermalize at low k -values. This difficulty can occur even if the pumping level is moderate, indicating that weak turbulence breaks down in such cases. Zakharov resolved this paradox through the discovery of the collapse process, which will be discussed in Sec. 6.

3. ZAKHAROV'S EQUATIONS

Zakharov [1972] started with the fluid equations for coupled Langmuir and ion acoustic waves and assumed two time scales: one characteristic of the acoustic waves and one for the Langmuir waves. Furthermore the ratio of electrostatic to thermal energy in the plasma was assumed to be less than the mass ratio:

$$W/n_0 T_e \ll \mu = m_e/m_i \quad (3)$$

With these two assumptions Zakharov found two coupled nonlinear equations for the evolution of the perturbed electric field E and the perturbed density n (we will use the dimensionless form of these equations) which are:

$$\begin{aligned} i \partial_t E + \partial_x^2 E &= nE \\ \partial_t^2 n - \partial_x^2 n &= \partial_x^2 |E|^2 \end{aligned} \quad (4)$$

For reference, if one wishes to convert back to dimensional variables, the transformations are as follows (the tildes over variables indicate the dimensionalized form) [Payne et al., 1984]:

$$\begin{aligned}
 t &= \left(\frac{2\eta}{3}\right) \mu \omega_{pe} \tilde{t} & n &= \left(\frac{3}{4\eta\mu}\right) \left(\frac{\tilde{n}}{n_0}\right) \\
 \omega &= \left(\frac{3}{2\eta}\right) \left(\frac{\tilde{\omega}}{\mu\omega_{pe}}\right) & E &= \left(\frac{1}{\eta}\right) \left(\frac{3\tilde{E}^2}{16\pi\mu n_0 T_e}\right)^{1/2} \\
 x &= \left(\frac{2}{3}\eta\mu\right) \left(\frac{\tilde{x}}{\lambda_{De}}\right) & \eta &= (\gamma_e T_e + \gamma_i T_i)/T_e
 \end{aligned} \tag{5}$$

where γ_e and γ_i are the adiabatic constants for electrons and ions.

The nonlinear coupling term on the right-hand side of the density equation is just the ponderomotive force associated with the perturbed electric field. When it is negligible, the left hand side can be Fourier analyzed to give the approximate ion acoustic dispersion relation. As we will see in later sections, when that electric field is strong, it can drive large density cavities (negative density perturbations) on the right hand side. When the term on the right hand side of the electric field equation is negligible, the left hand side can be Fourier analyzed to give the Langmuir dispersion relation. The nonlinear coupling term then represents the effect of the density perturbation on the plasma frequency in that dispersion relation.

Most of the research on Langmuir turbulence since 1972 has focused on these equations. They represent the first steps to the understanding of strong Langmuir turbulence, although some would object to the word "strong" because of the assumption made in Eq. (3). But these nonlinear equations are also very useful in analyzing some of the "quasi"-linear wave-wave interaction processes: that of parametric instabilities. Unlike the case of weak turbulence, where large numbers of waves are assumed to be interacting with random phases, parametric instabilities involve coherent interactions or the interaction of only a few waves. In the next section we will treat parametric instabilities using the Zakharov equations.

4. PARAMETRIC INSTABILITIES

Let us assume a pump wave that drives the instabilities that has an electric field of the form

$$E(x,t) = E_0 \exp[i(Kx - \Omega t)] \tag{6}$$

where E_0 is constant and real. In equilibrium (with no perturbations) this wave is taken to satisfy the linearized Zakharov equations, and

so $\Omega = K^2$. Assume that this field is large compared to the perturbation electric field associated with the Langmuir waves that it excites. Then the coupling terms on the right-hand side of Zakharov's equations are linear in this driving force and in the perturbation of the electric field E or density n :

$$\begin{aligned} (i\partial_t + \partial_x^2)E(x,t) &= nE_0 \exp[i(Kx - \Omega t)] \\ (\partial_t^2 - \partial_x^2)n(x,t) &= \partial_x^2 \{E_0 E \exp[i(Kx - \Omega t)] + E_0 E^* \exp[Kx - \Omega t]\} \end{aligned} \quad (7)$$

To solve these equations we could use a Floquet analysis as is done in Nishikawa [1968a,b], making use of the fact that the pump (driving) term has a characteristic periodicity in space and time. However, such a technique is not necessary for Eq. (7) since the actual solution is much simpler [Nicholson, 1983]. Notice that the right hand side of the first equation has the term $\exp[i(Kx - \Omega t)]$ as a factor. This drives a similar factor in the perturbation electric field E . Assume a solution of the form

$$E = \{A \exp[i(kx - \omega t)] + B \exp[-i(kx - \omega^* t)]\} \exp[i(Kx - \Omega t)] \quad (8)$$

When this is substituted into right-hand side of the second Zakharov equation, taking the absolute value rids the term of any dependence on the K and Ω but there is still a dependence on the k and ω . Thus the solution for the perturbed quantity n must have a dependence on those exponentials. Furthermore, the driving terms [the exponential of $i(kx - \omega t)$ and its complex conjugate] drive it with the amplitudes $E_0(A^* + B^*)$ and $E_0^*(A + B)$. Since the amplitudes are complex conjugates, we may write n in the complex conjugate form

$$n = \{C \exp[i(kx - \omega t)] + C^* \exp[-i(kx - \omega^* t)]\} \quad (9)$$

Substituting these forms of the solutions into Eq. (7) gives us an equation with coefficients of $\exp[i(kx - \omega t)]$ and $\exp[-i(kx - \omega^* t)]$. Equating the coefficients of each of these two terms independently gives an expression for the coefficients A and B in terms of C and C^* :

$$\begin{aligned} [(\Omega + \omega) - (K + k)^2]A &= -CE_0 \\ [(\Omega - \omega^*) - (K - k)^2]B &= -C^*E_0 \end{aligned} \quad (10)$$

Substituting into the second Zakharov equation we have an equation again with coefficients of the two types of \exp terms. We only need to keep the coefficients of the $\exp[i(kx - \omega t)]$ term to obtain the dispersion relation, since these terms only have C and not C^* :

$$\begin{aligned} -\omega^2 C + k^2 C &= Ck^2 E_0^2 \{1/[(\Omega + \omega) - (K + k)^2] \\ &+ 1/[(\Omega - \omega) - (K - k)^2]\} \end{aligned} \quad (11)$$

Eliminate the C and substitute for Ω from the relation $\Omega = K^2$ to yield the dispersion relation

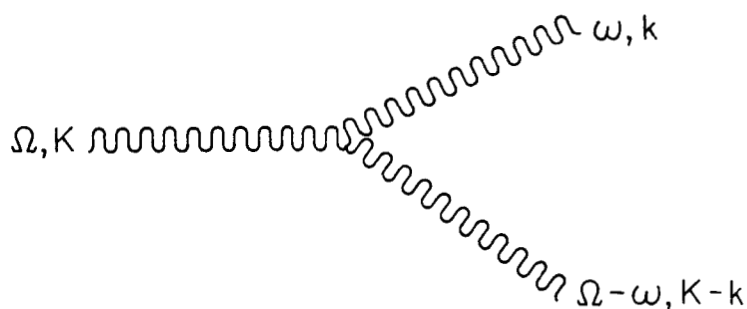
$$(\omega^2 - k^2) = k^2 E_0^2 \left[\frac{1}{(\omega - k^2 - 2Kk)} + \frac{1}{(-\omega - k^2 + 2Kk)} \right] \quad (12)$$

Eq. (12) is the dispersion relation for the parametric instability of the two terms on the right-hand side, one or the other can dominate. For example, if $|\omega|$ is small, the second term dominates, while if $|\omega|$ is large enough, the first term can dominate.

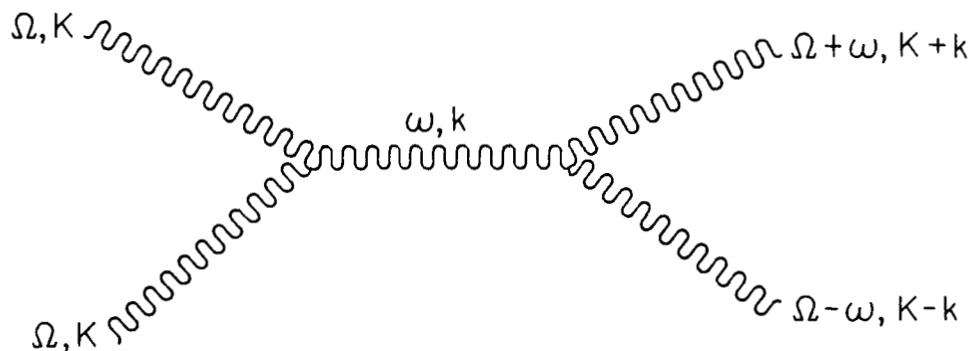
We will look for two types of solutions to the equations. These are indicated in Fig. 1. The first just involves a perturbation about

Fig. 1. Feynman diagram for the wave-wave interactions occurring in the parametric decay and oscillating two-stream instabilities.

B-G85-318



PARAMETRIC DECAY INSTABILITY



OSCILLATING TWO STREAM INSTABILITY

the $E_0 = 0$ solution $\omega = k$. Let $\omega = k + \delta$, with $|\delta| \ll k$. Substitute this into Eq. (12) to obtain the equation

$$\delta^2 + \delta(k - 2kK - k^2) - kE_0^2/2 = 0 \quad . \quad (13)$$

From the quadratic equation we see that this equation has unstable solutions provided that

$$E_0^2 > |k| (k + 2K - 1)^2/2 \quad (14)$$

This is basically a three wave interaction, and the instability a parametric decay. Thus the lowest threshold (and hence the greatest growth rate) occurs for $k = 1 - 2K$. Under this condition our growth rate becomes

$$\omega = i\gamma = i (|k|E_0^2/2)^{1/2} \quad (15)$$

The second type of solution is the oscillating two stream instability or purely growing mode. One can find this root in principle by substituting $\omega = i\gamma$ into Eq. (12) and solving the resulting fourth order algebraic equation for the real root γ . Such a procedure requires a computational solution. However, it is quite useful to make the dipole ($K=0$) assumption for the pump wave in this case, to simplify the algebra. In that case Eq. (12) becomes

$$(\gamma^2 + k^2)(\gamma^2 + k^4) - 2k^4E_0^2 = 0. \quad (16)$$

This can be solved for γ^2 by the quadratic equation:

$$\gamma^2 = \frac{1}{2} \{ (k^2 + k^4) \pm [(k^2 - k^4)^2 + 8k^4E_0^2]^{1/2} \} \quad (17)$$

Provided the $8k^4E_0^2$ term sufficiently large, the γ^2 root with the plus sign is positive and we have the purely growing mode. The threshold for this instability it is generally higher than that for the parametric decay. However, that is not always the case, as we shall see in Sec. 7.

5. ONE-DIMENSIONAL TURBULENCE -- ENVELOPE SOLITONS

In the last section we discussed the parametric instabilities driven by a constant pump wave. From the resulting coupled "linearized" equations, growth criteria and rates for the instabilities were determined. However, after a short period of time, two developments of the instability arise which invalidate this theory. First, the nonlinear coupling terms (associated only with the growing perturbations) increase in size and become important. Second, as the pump loses energy to the daughter waves its amplitude decreases. In this section this nonlinear stage of development is analyzed.

Let us consider the second Zakharov equation for the evolution of the density perturbation, Eq. (4). The temporal variations occur on the ion timescale. Provided $\partial_t^2 n \ll \partial_x^2 n$ we can neglect this term compared to the $\partial_x^2 n$ term. The resulting equation after a double spatial integration, setting the constants to zero, is

$$n = -|E|^2 \quad (18)$$

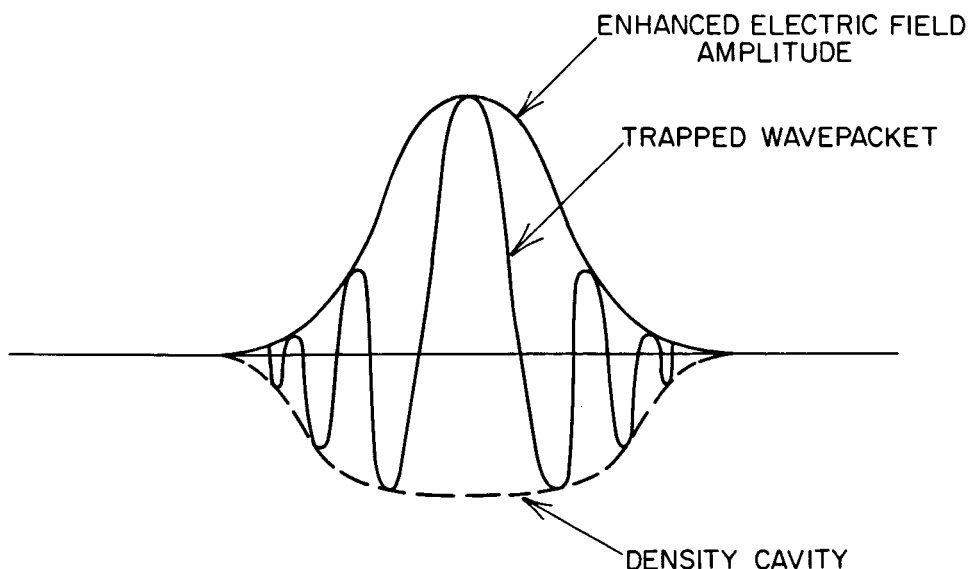
This has a simple physical interpretation. As the intensity of the electric field increases, and the resulting ponderomotive force which the right hand term represents also increases, the density perturbation becomes increasingly negative. Thus the localized wave-packet digs a density cavity (see Fig. 2). This cavity has the capacity to trap the wave packet. This can be understood more thoroughly from the first Zakharov equation, Eq. (4). The coupling term nE leads to contraction of the wavepacket; however, the other terms represent wave dispersion which tends to counteract this contraction.

Upon substitution of Eq. (18) into Eq. (4) we obtain the nonlinear Schrodinger equation:

$$(i\partial_t + \partial_x^2 + |E(x,t)|^2)E(x,t) = 0 \quad (19)$$

Fig. 2. Diagram of a trapped Langmuir wave packet. The regions of the large electric field is associated with the creation of a density cavity, which traps the packet.

B-G85-306



The nonlinear Schrodinger (NLS) equation for one-dimensional Langmuir turbulence was first examined by Zakharov and Shabat (1972). The approach used was that of inverse scattering theory. This technique was first used for the Korteweg de Vries (KdV) equation [Gardner et al., 1967] and is summarized in Fig. 3. The approach is in many ways analogous to the use of integral transform techniques for linear partial differential equations; in the latter the poles and branch cuts correspond to the scattering data which one then uses to aid in finding the inverse transform. The difficulty in the use of inverse scattering is that the inverse transform must be guessed, and there are few nonlinear equations for which those transforms are known. The KdV and NLS equations are cases for which the transforms have been found.

We will use a somewhat simpler approach to obtain a soliton solution. Any wavepacket that has a well-defined group velocity can be written in the form

$$E(x,t) = F(x-vt) \exp[i\phi(x-ut)] \quad (20)$$

where F and ϕ are real quantities, and v is the group velocity and u the group velocity. Substitute this into Eq. (19). All variables are real, so it is straightforward to separate out the real and imaginary parts:

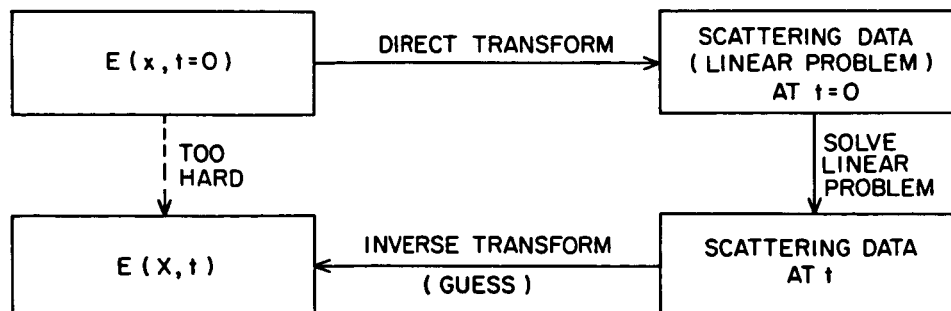
$$\begin{aligned} u\phi'F + F'' + -F(\phi')^2 + F^3 &= 0 \\ \phi''F + 2F'\phi' - vF &= 0 \end{aligned} \quad (21)$$

The second equation may be multiplied by F and integrated to give

$$F^2(\phi' - v/2) = C_1 \quad (22)$$

Fig. 3. Schematic of the inverse scattering method for solving certain nonlinear equations such as the nonlinear Schrodinger equation. The trick is to guess the inverse transform.

A-685-319



where C_1 is a constant of integration, which we will set equal to zero. This gives $\phi' = v/2$. Substitute this into the first equation we obtain

$$F'' - (v/2)^2 F + uvF/2 + F^3 = 0 \quad (23)$$

This equation may be multiplied by an integrating factor F' and integrated:

$$(F')^2 - (v^2/4 - uv/2)F^2 - F^4/2 = C_2 \quad (24)$$

where the second constant will also be set to zero. For symmetric solutions we will take $F'(0) = 0$, and denote $F(0) = F_0$. We can then solve for F_0 from Eq. (24):

$$F_0 = [v^2/2 - uv]^{1/2} \quad (25)$$

Eq. (24) can be integrated once again by taking the square root of both sides and separating the dependent variables on the right-hand side of the equation and the independent ones on the left hand side:

$$t = \int \frac{dF}{F[(v^2/2 - uv) - F^2]^{1/2}} \quad (26)$$

We will assume that $v > 2U$ in order to find a soliton solution. One can easily derive the standard form for the sech integral:

$$dx = \int \frac{dy}{y[1-y^2]^{1/2}} \quad (27)$$

where $y = \text{sech } x$. Using this and the initial condition in Eq. (25), we can find F and our solution:

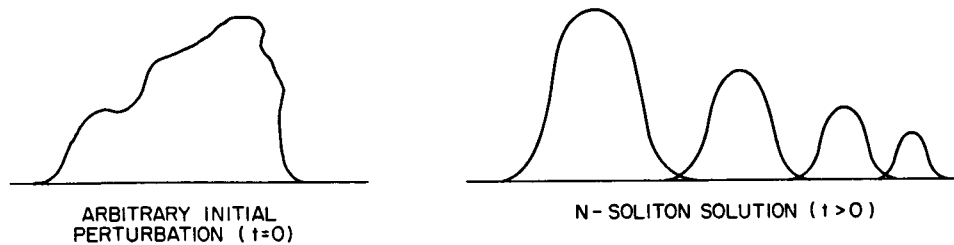
$$E(x,t) = F_0 \text{sech} [F_0(x-vt)/\sqrt{2}] \exp\{i[vx/2 - (v^2/4 - F_0^2/2)t]\}.$$

This is the soliton solution which arises because the tendency of the wavepacket to disperse is balanced by the trapping in the density cavity produced by the nonlinearity. This is shown schematically in Fig. 3.

This solution is a single soliton solution. In general the solutions to this equation are N -soliton solutions, where N is an integer number. The number of solitons excited depends on the strength of the initial perturbation. In the general case, one solves an initial value problem via the inverse scattering transform, and finds that any initial perturbation exhibits N -independent solitons which each show up individually after a period of time (see Fig. 4).

Fig. 4. Illustration of the general solution to the nonlinear Schrodinger equation which can be obtained by inverse scattering. Any arbitrary initial perturbation will evolve into N solitons.

B-G85-302

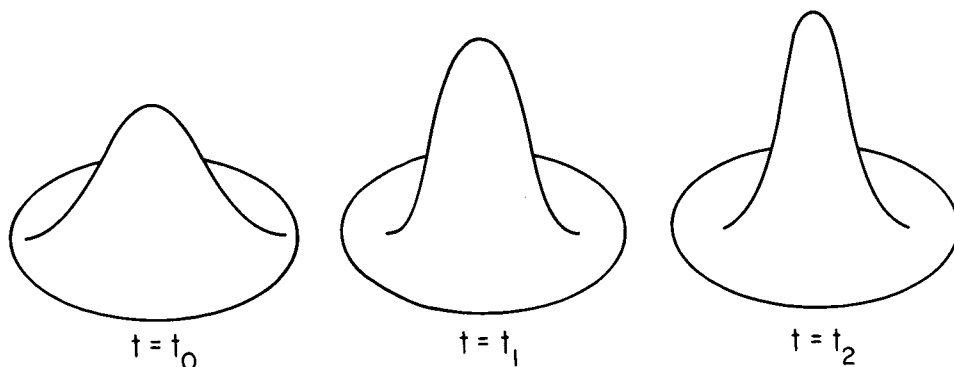


6. TWO AND THREE-DIMENSIONAL TURBULENCE -- WAVE COLLAPSE

In the last section we saw that for the one-dimensional case the nonlinear ponderomotive force, which is responsible for digging density cavities that trap the Langmuir wavepacket and thereby tend to compress it, just balances out the tendency of the wavepacket to spread out by dispersion. This allows for the existence of solitons, entities with long-term stability. However, as one goes to two and three dimensions, there is no reason to expect that the nonlinearity and the dispersion will be in perfect balance there as well. Thus one or the other effect may win out. We will show in this section that it is the nonlinear force that dominates over the dispersion, and that leads to collapse of the Langmuir wavepacket (see Fig. 5). This collapsing wavepacket is the nonlinear stage of the oscillating two-stream instability, also called the modulational instability.

Fig. 5. Perspective of the self-similar collapse of a Langmuir soliton in two and three-dimensional theory.

B-G85-303



The full three-dimensional Zakharov equations are

$$\begin{aligned} i\partial_t(\nabla \cdot \vec{E}) + \nabla^2(\nabla \cdot \vec{E}) &= \nabla \cdot (n\vec{E}) \\ \partial_t^2 n - \nabla^2 n &= \nabla^2 |\vec{E}|^2 \end{aligned} \quad (28)$$

To solve these we look for solutions involving self-similar collapse [Zakharov, 1972; Galeev, et al., 1975; Weatherall et al., 1982; Nicholson, private communication]:

$$\begin{aligned} \vec{E} &= (t_0 - t)^\alpha \vec{F}(\vec{y}) \exp(i \int u dt) \\ n &= (t_0 - t)^\beta \vec{G}(\vec{y}) \\ \vec{y} &= (t_0 - t)^\delta \vec{x} \\ u &= C(t_0 - t)^\epsilon \end{aligned}$$

We may substitute these expressions into the original equations. The evaluation of the derivatives is tedious but straightforward. We may note, for example, that

$$\begin{aligned} \partial_t \vec{F}(\vec{y}) &= \vec{F}'(\vec{y}) \cdot d\vec{y}/dt \\ &= - (t_0 - t)^{-1} \vec{y} \cdot \vec{F}'(\vec{y}) \end{aligned} \quad (30)$$

Each term in the Zakharov equations similarly has $(t_0 - t)$ to some order associated with it. The first equation has terms

$$\begin{aligned} i\partial_t \vec{E} &\sim (t_0 - t)^{\alpha-1}, (t_0 - t)^{\alpha+\epsilon} \\ \partial_x^2 \vec{E} &\sim (t_0 - t)^{\alpha+2\beta} \\ n\vec{E} &\sim (t_0 - t)^{\alpha+\delta}, \end{aligned} \quad (31)$$

and setting them all to the same power gives the relationship

$$\epsilon = 2\beta = \delta = -1. \quad (32)$$

The second equation similar has terms like

$$\begin{aligned}
\partial_t^2 n &\sim (t_0 - t)^{\delta-2} \\
\partial_x^2 n &\sim (t_0 - t)^{\delta+2\beta} \\
\partial_x^2 |E|^2 &\sim (t_0 - t)^{2\alpha+2\beta}
\end{aligned}
\tag{33}$$

We now ask the question as to whether it is possible for all of these terms to be of the same order. If this is to be the case, then we have from Eq. (33)

$$\beta = -1 \tag{34}$$

However, this is inconsistent with the first equation, Eq. (32). Thus it is not possible to have all of the terms of the same order.

Zakharov [1972] use the subsonic approximation, such that the temporal variations vary much slower than the spatial variations: $\partial_t^2 n \ll \partial_x^2 n$. This is reasonable because n varies on the slow acoustic time scale. Substituting the self-similar solutions into the second Zakharov equation with the time derivative term ignored gives:

$$\delta = 2\alpha \tag{35}$$

With the rest of the terms equated, Eqs. (32) and (35) give the final solution for the exponents in Eq. (29) as:

$$\alpha = \beta = -1/2; \quad \delta = \epsilon = -1 \tag{36}$$

This is a solution for the collapse of the wavepacket. However, as the wavepacket collapses a point may be reached where the subsonic approximation is not valid. Weatherall et al. [1982] have shown that it is good for weaker fields so that $W/n_0 T_e \ll m_e/m_i$, while for very intense fields with $W/n_0 T_e \gg m_e/m_i$ the collapse is supersonic (where the time derivative term dominates over the spatial derivative term). In the latter case a different set of exponents is obtained for the self-similar collapse.

It is clear that these types of solutions collapse to a singularity in a finite time. This has sometimes been referred to as the "black hole" of plasma physics. However, before that state is reached, the Zakharov equations break down. What happens in these late stages of collapse? Zakharov [1972] refers to this point of breakdown as the "intersection of electron trajectories". This is basically the intersection of wave characteristics, or wavebreaking. An analysis of this stage is necessary in order to further understand the nature of Langmuir turbulence.

7. IONOSPHERIC MODIFICATION

Probably the oldest area of space plasma physics studies is that of radio wave propagation in the ionosphere. This is an area of active experimentation, and in recent years the research has concentrated on what happens at high transmitter power levels. High power ionospheric heating experiments in the United States were initiated in 1970 at the Platteville, Colorado, and the Arecibo, Puerto Rico facilities. These experiments use a HF heating wave at about 5-10 MHz. In addition, at Arecibo a UHF diagnostic wave at 430 MHz is used. This allows for the study of some interesting nonlinear effects which can lead to significant modifications of the densities and the temperatures of the plasma there. The predominate phenomena that occur are parametric instabilities, nonlinear absorption, and possible soliton formation. In addition to being a laboratory for the study of nonlinear plasma effects, much can be learned about the properties of the ionosphere. For excellent reviews on this subject see Perkins et al. [1974], Fejer [1977], and Fejer [1979].

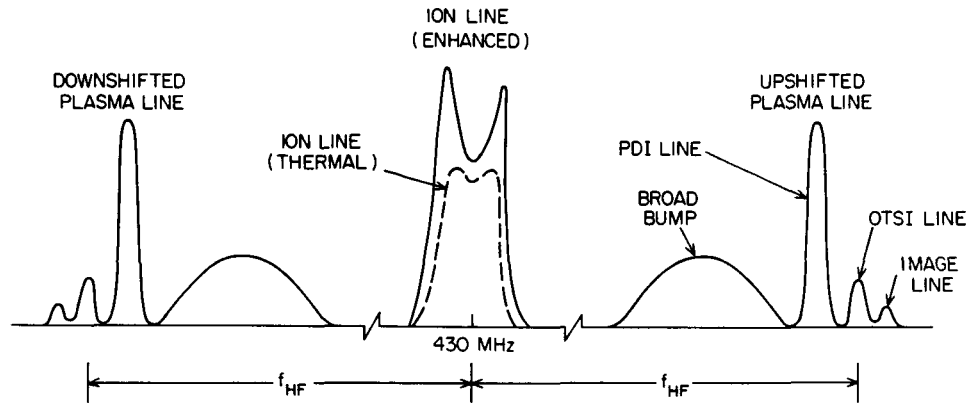
Modifications in the ionosphere of particular interest in the study of Langmuir turbulence occur near the F region. That is the region where the ionospheric density becomes sufficiently high that the wave frequency is below the plasma frequency, and the wave reflects. Near the reflection point the wave frequency is just above the plasma frequency, and parametric instabilities are possible in which the electromagnetic wave excites Langmuir waves. One of the first observations was that of so-called "artificial spread-F", where instead of a single reflection layer many closely-spaced reflection layers were observed to appear shortly after the transmitter was turned on [Utlaut et al., 1970; Utlaut and Violett, 1974]. This was evidenced by the fact that the reflection echoes of the transmitted wave had several different time delays, rather than a single one. This phenomena was observed to be independent of the mode of propagation.

Two theories have been proposed for the origin of the artificial spread-F. The first involves something like the oscillating two stream instability [Cragin and Fejer, 1974; Cragin et al., 1977]. However, in this approach the nonlinear force is a collisional dissipation term rather than the ponderomotive force term. The second approach involves a thermal self-focusing instability arising from collisional heating [Perkins and Valeo, 1974]. This approach is similar to the modulational instability. However, instead of the ponderomotive force creating refraction index increases and thus wave concentration, what happens is that the wave-induced heating increases the temperature, which causes a hydrodynamic expansion and a simultaneous increase in the index of refraction, and concentrates the wave.

The type of backscattered spectrum that is typically observed during heating experiments is indicated in Fig. 6 (see for example, Fejer [1977]). The UHF diagnostic wave is sent up at 430 MHz, and the

Fig. 6. Typical incoherent backscatter spectrum obtained from the ionospheric heating facility in Arecibo, Puerto Rico. This uses a 430 MHz diagnostic wave and a high frequency heating wave in the range of 5-10 Hz.

C-685-304



scattering spectrum has a frequency band around this frequency, as well as downshifted and upshifted plasma lines. The various aspects of this spectrum will be explained in the following paragraphs.

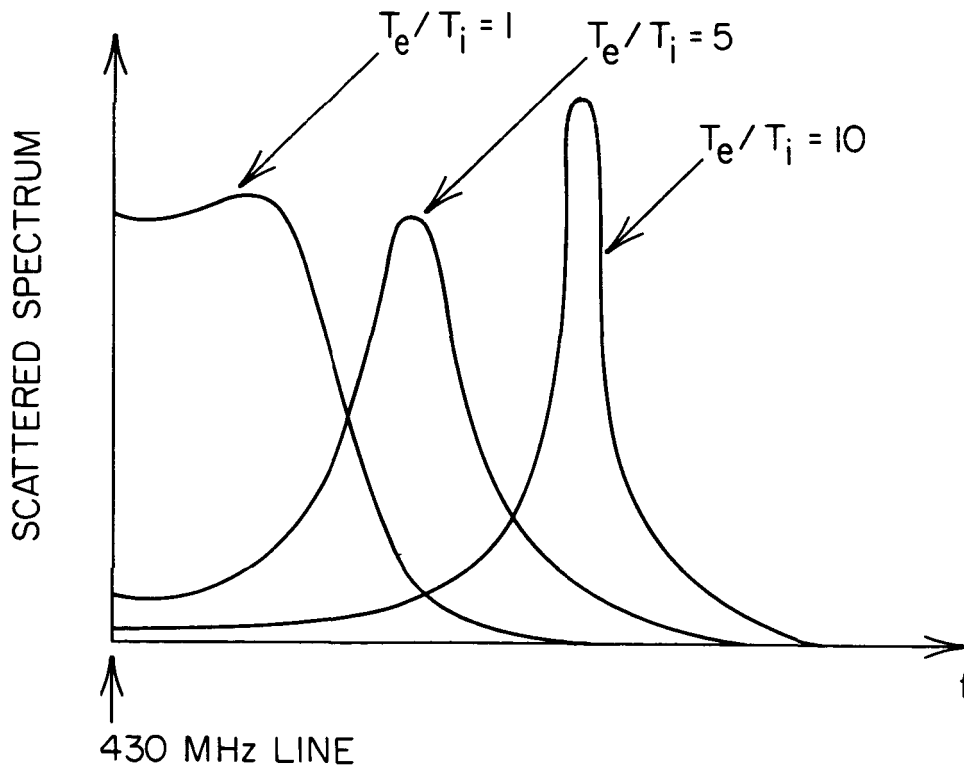
The phenomenon of the broadening of a transmitted wave upon backscatter is explained quite well by the dressed test particle theory of Rosenbluth and Rostoker [1962]. In the early days it was thought that the broadening would be determined by the electron thermal velocity so the shoulders would occur at $\omega \pm kv_e$, since the electron response produces the backscatter. However, experiments in the early '60's had shown the broadening of low power radio backscattering to be determined by the ion thermal velocity, with the shoulders being at $\omega \pm kv_i$. This was a somewhat surprising result at the time. The basic explanation of this phenomenon was provided by Dougherty and Farley [1960], Fejer [1960], and Salpeter [1960], who showed that for long wavelengths the response should be at the ion thermal velocity, while for short wavelengths it should be at the electron thermal velocity. Shortly afterwards Rosenbluth and Rostoker developed the full dressed test particle theory of the radiative response of the plasma. Because the wavelength of the wave is much longer than a Debye length of the plasma, it is the collective effects, and not free electrons that determine this broadening. If one considers responses of the shielded test electron, one finds in the shielding cloud one missing electron and thus the cloud as a whole contains no net electron fluctuations to scatter the radiation. A shielded ion cloud, however, has an excess half electron and a shortage of half an ion, and it is these excess electrons that scatter the radiation. But these electrons can only respond according to the characteristic thermal velocity of the test ion of the cloud they are in. This explains the low thermal

broadening, and also why the total scattering is only half of what is expected from free electron plasmas.

This is not the whole picture, however. When the high power heaters started up, it was noticed that the ion line signal became enhanced, where the signal would become broader, and the intensity at the shoulders would become much sharper (compare the enhanced line with that obtained without the enhancement in Fig. 6). This is explained quite well by Bernstein et al. [1964]. The result that the shoulder position is determined by the ion thermal velocity is only correct for equal ion and electron temperatures. When the heater is on, it heats primarily the electrons, producing electron temperatures greater than the ion temperatures. Under these cases the ion acoustic waves are strongly excited, and they are responsible for the frequency shifting to produce the shoulders. The effects of the acoustic waves dominate over the effects of the dressed test ions when $T_e \gg T_i$ (see Fig. 7).

Fig. 7. The shoulder formation expected in the ion line of the backscatter spectrum for various temperature ratios. When $T_e \gg T_i$ the ion acoustic waves are responsible for this broadening [after Bernstein et al., 1964].

B-G85-305



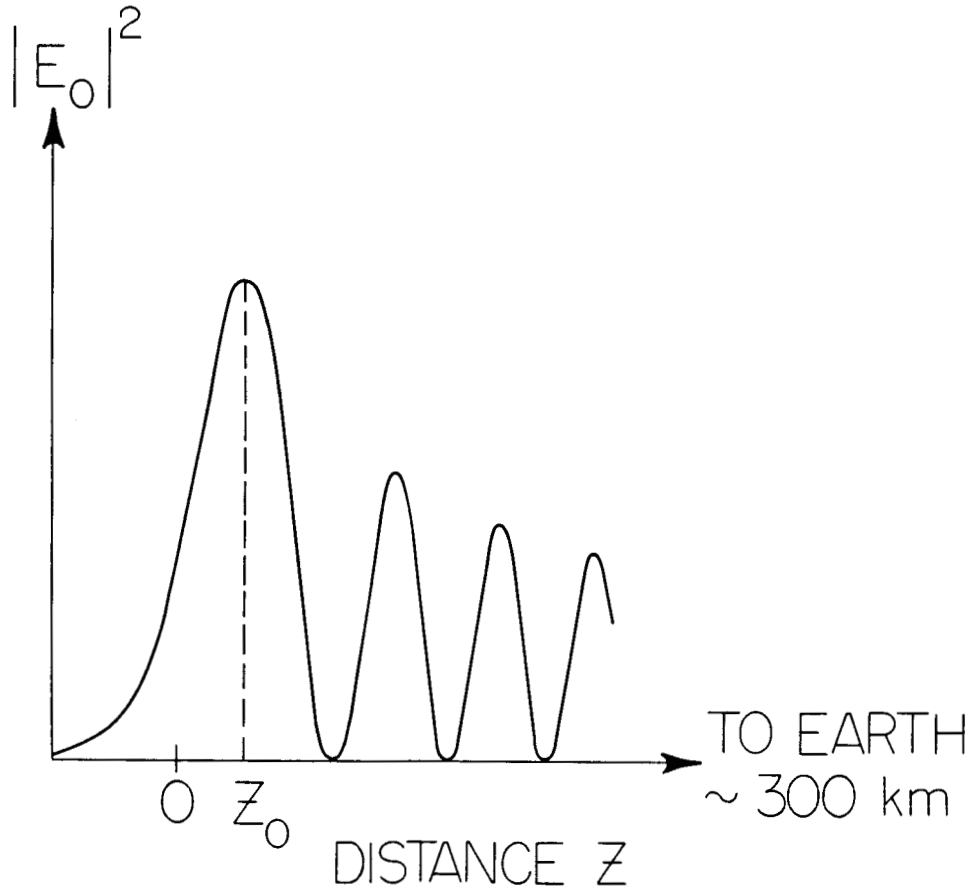
The sharp low-frequency and high-frequency peaks in Fig. 6 correspond to the upshifting and downshifting of the 430 MHz wave by the HF wave. The HF wave excites upgoing and downgoing plasma waves through a parametric decay or oscillating two-stream instability. (The fact that the oscillating two-stream instability would be excited by the heating transmitter was first predicted by Perkins and Kaw [1971].) The 430 MHz wave then scatters off the plasma waves, being downshifted by the upgoing waves and upshifted by the downgoing waves. The up and downshifted peaks that are just below the HF lines correspond to the parametric decay instability, while those that are at the HF frequency are due to the oscillating two-stream instability (in the decay instability a downshifting of the HF frequency by an ion acoustic frequency occurs). In addition, the plasma waves that are excited by the decay instabilities can undergo further decays by a sequence of emissions of ion acoustic waves, and these plasma waves that have cascaded down interact with the 430 MHz backscattering signal to produce the "broad bumps" observed. In addition, the decay wave can interact with the oscillating two-stream (OTS) wave to produce a small decay image line that appears displaced from the OTS line by the same frequency that the OTS line is displaced from the PDI line. It was also shown recently that these parametric instabilities do not have to be above threshold for the plasma lines to be present. Fejer and Sulzer [1984] have shown that stable fluctuations excited by the heater wave can interact with the ion line and still produce an observable line. For an in-depth analysis of the theory of these parametric instabilities see Perkins et al. [1974].

In the diagram the peaks of the OTS are lower than those of the decay instability. However, this is not always the case. Under certain conditions the OTS may have a lower threshold for instability than for the PDI. This can occur in the ionosphere since under normal conditions ion and electron temperatures are equal there, making ion acoustic waves highly damped and shutting down this decay channel for the decay instability. The question of the onset threshold of these instabilities in the ionosphere has been examined by Payne et al. [1984], and Nicholson et al. [1984]. Data supporting this fact has been reported by Fejer et al. [1983], where the use of a 47 MHz diagnostic radar in combination with a heater showed no evidence of an upshifted parametric decay line. The theoretical work mentioned explains why only a OTS instability line occurs. The authors show that at the point of maximum electric field in the ionosphere (the maximum of the Airy function form of the electromagnetic wave in the inhomogeneous plasma, as shown in Fig. 8, the threshold for the OTS or modulational instability is exceeded, and the growth rate is much faster than the PDI. This instability theoretically should lead to rapid formation of solitons.

Studies have similarly been carried out at the exact classical reflection points (where the transmitter frequency equals the plasma frequency), and it has also been found that soliton formation and subsequent three-dimensional collapse occurs there as well [Sheerin et

Fig. 8. Standing wave pattern of the heater wave in the inhomogeneous ionosphere. For a linear density gradient this is an Airy function. $Z=0$ is the exact reflection height in the ionosphere, and $Z=Z_0$ the position of the Airy maximum [after Weatherall et al., 1982].

C-G81-223-1



al., 1982; Weatherall, 1982], as only the oscillating two-stream instability occurs for a pump frequency exactly equal to the plasma frequency (no decay is possible). These studies find a possible explanation for another phenomenon known as plasma line overshoot. This occurs just after the heater is switched on after being off for some time. The HF enhanced plasma line produced by the decay mode rises to a large amplitude then drops back after a few tens of milliseconds to its steady state value. The reason for this overshoot could be explained by the initiation of three-dimensional collapse, which would temporarily give a significant rise in the amplitude until a steady state situation is reached [Duncan and Sheerin, 1985]. This is by no means the only explanation of this phenomenon, but it is an intriguing one [Muldrew, 1985].

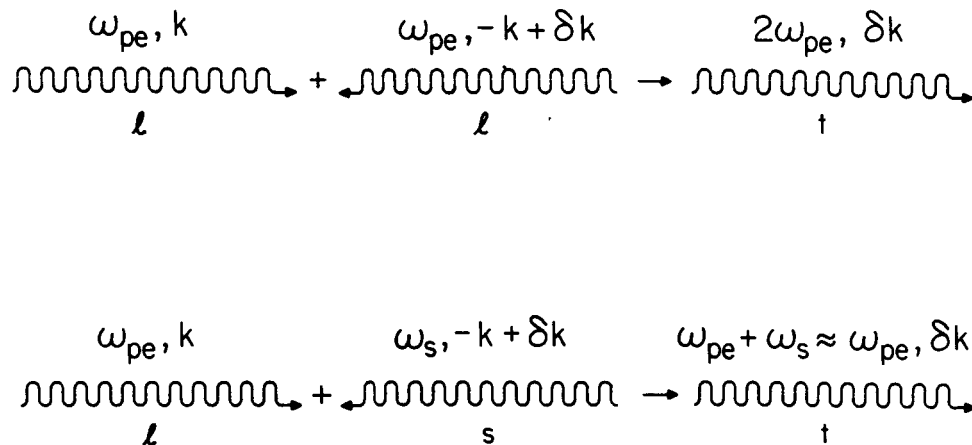
8. TYPE-III SOLAR RADIO BURSTS

The Sun and its environment are responsible for a number of radio wave emissions. These are classified according to the types or character of the bursts. Type-III bursts are generated during solar flares, and have the characteristic that the frequency of emission decreases with time, usually on the time scale of a few hours. They were discovered by Wild [1950], who noted that they were generated near the local plasma frequency f_{pe} . The frequency decrease with time is explained by the fact that particles ejected and accelerated by the solar flare trigger the emission of the bursts and the local plasma density falls with increasing distance from the Sun.

A model for the generation of these bursts was proposed by Ginzburg and Zheleznyakov [1958] which invoked weak turbulence theory for Langmuir waves. In this model the streaming electrons in the flare excite the Langmuir waves by a bump-on-tail instability. The excited Langmuir waves are then converted into transverse waves by nonlinear wave-wave interactions. The latter process involves the $\ell + \ell \rightarrow t$ conversion discussed in Section II. There are two ways this process can work in conjunction with Langmuir waves: two Langmuir waves with k -vectors pointing in opposite directions can interact to produce a transverse wave with a frequency at twice the plasma frequency and k near zero, or a Langmuir wave can interact with an ion acoustic wave to produce a transverse wave at the plasma frequency. (See Fig. 9).

Fig. 9. Schematic of the Langmuir wave interactions that can occur in weak turbulence theory to produce transverse waves. The first interaction is $\ell + \ell \rightarrow t$ at twice the plasma frequency, and the second is $\ell + s \rightarrow t$ at the plasma frequency.

A-G85-320



Evidence was found for the association of Langmuir waves with the type III bursts by Gurnett and Anderson [1976] and Gurnett et al. [1980]. They showed that components of the bursts were observed both near the plasma frequency and at the second harmonic at the same time that intense Langmuir waves were observed. Although this appeared to support the weak turbulence mechanism proposed in 1958, theoretical work on the problem had advanced well beyond this early model.

The problem of the nonlinear development and saturation of the linear beam instability was first addressed by Zheleznyakov and Zaitsev [1970]. They concluded that quasilinear effects set in and cause deceleration of the beam by plateau formation in velocity space before any nonlinear process becomes important. This result is problematic for the model, however, because this would mean the streams would be decelerated within a few hundred kilometers. Yet observations show that the source region for the bursts travels at almost constant velocity from the inner corona out to almost 1 A.U.

Papadopoulos et al. [1974] conclude, however, that instead of saturation by quasilinear diffusion, which would cause deceleration of the beams and plateau formation, the beams stabilize by the oscillating two stream instability discussed in Section IV. This could explain the observations on the persistence of the beam velocities. It was shown that the efficiency of the oscillating two-stream instability for converting Langmuir waves into electromagnetic waves was sufficiently high so that the observed power levels could be achieved. With the evolution of this oscillating two-stream instability for the one-dimensional case into the nonlinear regime, the Langmuir waves could be predicted to evolve into solitons. Papadopoulos and Freund [1978] examined the radio emission that would be produced by these Langmuir solitons and showed that they could produce a radiation spectrum at a frequency of twice the plasma frequency similar to what was observed.

The analysis of this problem continued with Bardwell and Goldman [1976]. The previous work had been one-dimensional, but they used a three-dimensional model with cylindrical symmetry. In this more general treatment they investigated the oscillating two-stream (OTS) instability, the backscattering of the waves off of ions, and in addition a new modulational instability which they called the stimulated modulational instability (SMI). Both the OTS and the SMI involve the same type of 4-wave interaction (see Fig. 1). However, the SMI is similar to the parametric decay instability, except that 4 waves are necessary for the k matching to occur. The conclusion was that the oscillating two stream instability probably cannot saturate the beam instability, and would not prevent the deceleration and plateau formation of the beam. Thus the parametric instabilities would not resolve the problem of the persistence of the beam velocity.

Nicholson et al. [1978] examined the problem the nonlinear evolution of the Langmuir waves numerically. This work supported the

conclusion drawn from the one-dimensional studies that nonlinear effects were more important than the quasilinear effects. However, a new nonlinear phenomenon was found to occur for the conditions at the source region: two-dimensional soliton collapse (which was discussed in Sec. 6). This collapse was found to produce a much broader range of wave spectra in k-space than could be produced by quasilinear theory, in agreement with observation. This resolves the problem of quasilinear deceleration of the beam as well.

The question of whether the interactions leading to type III bursts are one-dimensional has never been fully settled. In a series of papers by Smith et al. [1979] and Goldstein et al. [1979] an extensive analysis of the three-dimensional equations describing the nonlinear interactions was made, and the conclusion drawn that for the conditions in the source region for type III bursts, they reduced to a one-dimensional description because of the local magnetic field. The results of the analysis were applied to bursts observed near 1 A.U. For those conditions it was shown that the OTS instability and anomalous resistivity (created by the Langmuir turbulence) cause a rapid transfer of the energy to short wavelengths, out of resonance with the beam. Thus quasilinear effects cannot slow the beam. A prediction of the theory involved the scaling of the radio wave flux with the electron flux of the flare; the theory was shown to give good agreement with observations.

The alternative explanation for the reason the beams do not slow down is reviewed by Goldman [1983]. Because there is velocity dispersion in the beams, at a given point fast electrons arrive first and slow ones come along behind. As that happens the bump on the tail of the velocity distribution function moves to lower velocities, and some of the waves generated by the fast electrons become reabsorbed because where the slope was originally negative it is now positive. This would allow for long-distance propagation without slowing, and is consistent with quasi-linear saturation.

The spectrum of Langmuir waves associated with type III burst are observed to be quite spiky, with the peak amplitude of the spikes several orders of magnitude above the mean amplitude of the waves. An analysis of these data implied that these spikes were likely not associated with the collapsing solitons discussed by Nicholson et al., but were instead due to plasma wave clumping created by large density inhomogeneities [Smith and Sime, 1979]. A review is given of the whole problem of nonlinear effects involved in the generation of the type III bursts, including the problems of weak and strong turbulence radiations, soliton collapse, and plasma wave clumping, in Smith [1979].

In summary, it looks rather likely that type III solar bursts are generated from second harmonic, and possibly even first harmonic emission, probably from Langmuir wave collapse. For an excellent review of the theoretical and experimental situation, see Goldman

[1983]. In addition, a brief review of the topic of type III bursts is contained in Papadopoulos [1979].

9. LANGMUIR TURBULENCE UPSTREAM OF THE BOW SHOCK

Not only does Langmuir turbulence occur in the solar wind associated with solar flares, but when the solar wind interacts with planetary magnetic fields it can be produced. Langmuir waves have been observed upstream of both the Earth's and Jupiter's bow shocks. Of particular interest is the case of the Jovian bow shock, where a transition to turbulence is seen [Gurnett et al., 1981]. The Langmuir waves are generated by electron beams (by the gentle bump instability) arriving from a tangential point off the bow shock (see Fig. 10). As they are generated they are carried back toward the bow shock by the solar wind. One can then study the evolution of the waves by observing the characteristics as a function of the distance from the beam that generated them. What is found is that the waves near the beam show a clear narrowly-spaced frequency spectrum at the local plasma frequency (see Fig. 11). However, somewhat further from the beam they show evidence of turbulence, being spread out in frequency with many spikes, which are probably due to sideband emissions due to ion acoustic waves or other low frequency noise. Even further away they show strong spreading and strong spikes. The last two cases clearly represent nonlinear regimes of the Langmuir waves.

Fig. 10. The generation of Langmuir waves upstream of the Earth's bow shock. The electron beam that is tangent to the shock generates the waves, which are carried back toward the shock by the solar wind. Linear and nonlinear regimes of the waves are indicated [after Gurnett et al., 1981].

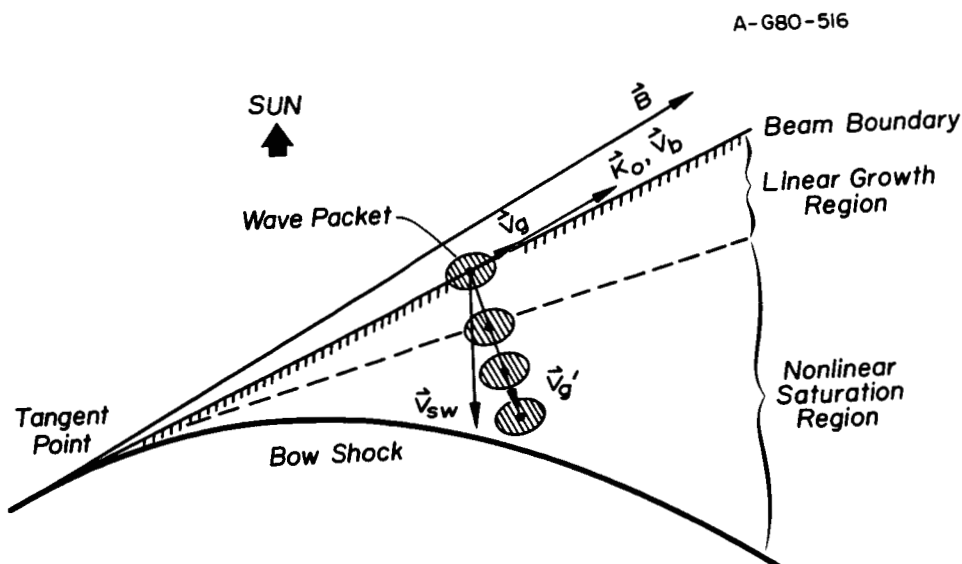
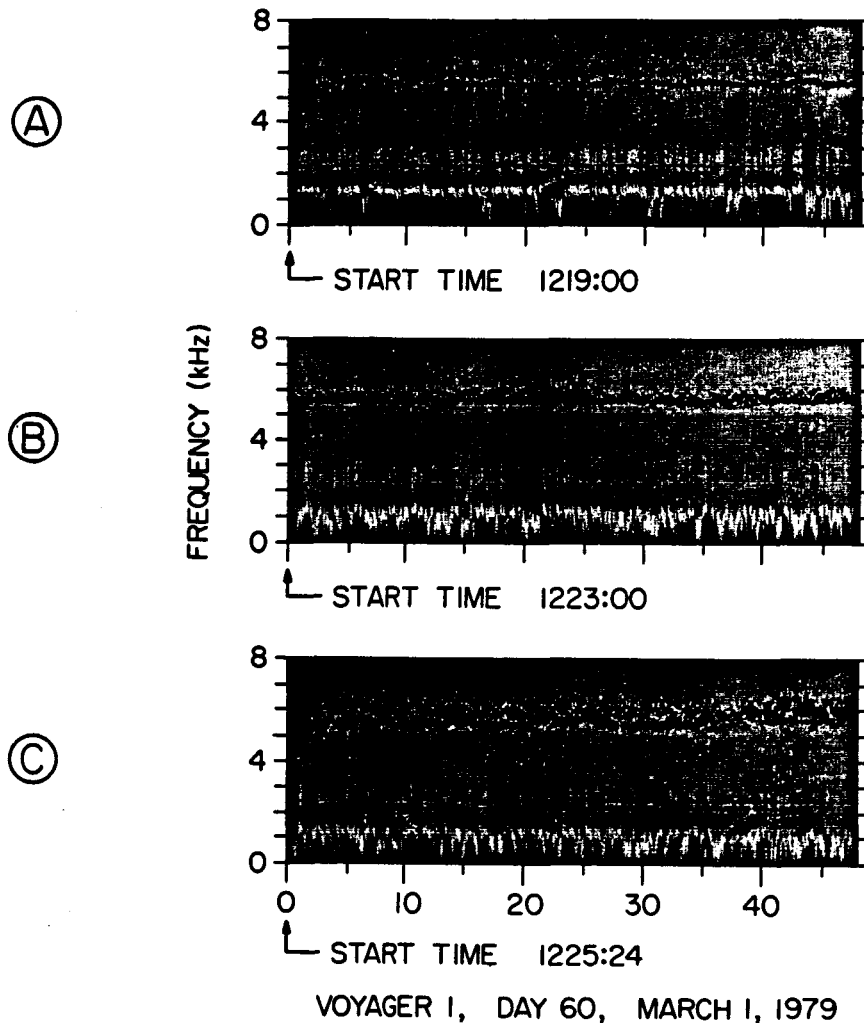


Fig. 11. Wave data on the evolution of the plasma turbulence in the upstream region. The top diagram shows linear Langmuir waves. The second diagram shows strong broadening and upper wideband generation, indicating significant nonlinear interactions. The last diagram shows the even stronger broadening and sideband generation, indicating a very turbulent state [after Gurnett et al., 1981].



The interpretation for the evolution into turbulence involves parametric instabilities. Because the spiky turbulence showed sidebands at frequencies higher than that of the principle Langmuir waves, this was interpreted as being a result of the oscillating two-stream instability. This is as opposed to the parametric decay instability, which principally only produces lower sidebands, as discussed in Sec. 4. In addition, Gurnett et al. provided some evidence for short-term large-amplitude Langmuir wave bursts which they interpreted as possibly being collapsing solitons. However, no convincing evidence was found that they indeed were collapsing.

A different interpretation of the nonlinear mechanisms involved in this Langmuir turbulence was provided by Russell and Goldman [1983]. Utilizing the observations of density fluctuations simultaneous to the observations of the electron beams and Langmuir turbulence, they investigated the influence of the fluctuations on the generation of the Langmuir turbulence by the bump-in-tail instability. Multiple backscatterings of the Langmuir waves off the density fluctuations can occur. In the process the waves are upshifted to higher wavenumbers, and thus to higher frequencies. This continues until the waves reach sufficiently large k -values that they are either dissipated by Landau damping or shifted to a wavenumber higher than cutoff. It was shown that the observed level of density fluctuations is sufficient to saturate the unstable Langmuir waves, providing for the saturated spectrum of Langmuir turbulence observed.

10. SUMMARY AND CONCLUSIONS

We have discussed three major applications of plasma turbulence in the space plasma area: to active experimental studies in the ionosphere, and to observations of solar radio bursts and waves upstream of the bow shock. This by no means exhausts the list, because experimental and theoretical investigations are indicating the possible importance of this kind of turbulence to other areas in space and astrophysical plasmas. Two examples will be mentioned. The first is the observation of 3 kHz radiation in the outer heliosphere (outer part of the solar system) [Kurth et al., 1984]. This occurs right above the plasma frequency in the solar wind. Thus it can only have been generated by plasma turbulence out at that distance. The second are some theoretical papers ascribing some of the features of pulsar radiation fields to Langmuir solitons and their radiation [Buti, 1978; Melikidze and Pataraya, 1980]. These developments may be indicative that the regions in space where Langmuir turbulence is known to be important are expanding in number. The development of our understanding of the basic physics involved in the turbulence is far from exhausted, and its widespread application to the understanding of space physics and astrophysics may in many ways be just beginning.

11. ACKNOWLEDGEMENTS

We would like to thank Dwight Nicholson for a critical reading of the manuscript. This work was sponsored by grants from the National Science Foundation and the National Aeronautics and Space Administration of the United States.

12. REFERENCES

- Bardwell, S., and Goldman, M.V., "Three-Dimensional Langmuir Wave Instabilities in Type III Solar Radio Bursts," *Ap. J.* 209, 912-926 (1976).
- Bernstein, I., Trehan, S., and Weenick, M., "Plasma Oscillations: II: Kinetic Theory of Waves in Plasmas," *Nucl. Fusion* 4, 61-104 (1964).
- Cragin, B.L., and Fejer, J.A., "Generation of Large-Scale Field-Aligned Irregularities in Ionospheric Modification Experiments," *Radio Sci.* 9, 1071-1075 (1974).
- Cragin, B.L., Fejer, J.A., and Leer, E., "Generation of Artificial Spread F by a Collisionally Coupled Purely Growing Parametric Instability," *Radio Sci.* 12, 273-284 (1977).
- Dougherty, J.P., and Farley, D.T., "A Theory of Incoherent Scattering of Radio Waves by a Plasma", *Proc. Roy. Soc. Lond. Ser. A* 259, 79 (1960).
- Duncan, L.M., and Sheerin, J.P., "High Resolution Studies of the HF Ionospheric Modification Interaction Region", *J. Geophys. Res.* (1985, in press).
- Fejer, J.A., "Parametric Instabilities in the Ionosphere," *J. Phys. (Paris)* 38, C6-55 (1977).
- Fejer, J.A., "Scattering of Radio Waves by an Ionized Gas in Thermal Equilibrium", *Can. J. Phys.* 38, 1114 (1960).
- Fejer, J.A., "Ionospheric Modification and Parametric Instabilities", *Rev. Geophys. Space Phys.* 17, 135-53 (1979).
- Fejer, J.A., Ierkic, H.M., Woodman, R.F., Rottger, J., Sultzer, M., Behuke, R.A., and Veldhuis, A., "Observations of the HF-Enhanced Plasma Line with a 46.8 MHz Radar and Reinterpretation of Previous Observations with the 430 MHz Radar", *J. Geophys. Res.* 88, 2083-92 (1983).
- Fejer, J.A. and Sulzer, M., "The HF-Induced Plasma Line Below Threshold", *Rad. Sci.* 19, 675 (1984).
- Galeev, A.A., Sagdeev, R.Z., Sigov, Y.S., Shapiro, V.D., and Shevchenko, V.I., "Nonlinear Theory for the Modulation Instability of Plasma Waves", *Sov. J. Plasma Phys.* 1, 5-10 (1975); *Fiz. Plasmy* 1, 10-20 (1975).

- Nishikawa, K., "Parametric Excitation of Coupled Waves. I. General Formulation, II. Parametric Plasmon-Photon Interaction", J. Phys. Soc. Jpn. 24, 916-922 and 1152-1158 (1968).
- Papadopoulos, K., "Interplanetary Type III Radiobursts", Rev. Geophys. Space Phys. 17, 624-6 (1979).
- Papadopoulos, K., Goldstein, M.L., and Smith R.A., "Stabilization of Electron Streams in Type III Solar Radio Bursts," Ap. J. 190, 175-85 (1974).
- Papadopoulos, K. and Freund, H.P., "Solitons and Second Harmonic Radiation in Type III Bursts," Geophys. Res. Lett. 5, 881-4 (1978).
- Payne, G.L., Nicholson, D.R., Downie, R.M., and Sheerin, J.P., "Modulational Instability and Soliton Formation during Ionospheric Heating", J. Geophys. Res. 89, 10,921-28 (1984).
- Perkins, F.W., Oberman, C., and E.J. Valeo, "Parametric Instabilities and Ionospheric Modification", J. Geophys. Res. 79, 1478-96 (1974).
- Perkins, F.W., and Valeo, E.J., "Thermal Self-Focusing of Electromagnetic Waves in Plasma", Phys. Rev. Lett. 32, 1234-37 (1974).
- Rosenbluth, M.N., and Rostoker, N., "Scattering of Electromagnetic Wave by a Nonequilibrium Plasma", Phys. Fluids 5, 776-88 (1962).
- Rudakov, L.I., and Tsytovich, V.N., "Strong Langmuir Turbulence", Phys. Reports 40 1-73 (1978).
- Salpeter, E.E., "Electron Density Fluctuations in a Plasma", Phys. Rev. 120, 1528 (1960).
- Sheerin, J.P., Weatherall, J.C., Nicholson, D.R., Payne, G.L., Goldman, M.V., and Hansen P.J., "Solitons and Ionospheric Modification", J. Atm. Terr. Phys., 44, 1043-1048 (1982).
- Smith, D.F., and Nicholson, D.R., "Nonlinear Effects Involved in the Generation of Type III Solar Radio Bursts", pp. 225-243 in P.J. Palmadesso and K. Papadopoulos (eds.), Waves and Instabilities in Space Plasmas, D. Reidel, Holland (1979).
- Smith, R.A., Goldstein, M.L., and Papadopoulos, K., "Nonlinear Stability of Solar Type III Bursts. I. Theory", Ap. J. 234, 348-62 (1979).
- Ter Haar, D. and Tsytovich, V.N., "Modulation Instabilities in Astrophysics", Phys. Reports 73, 175-236 (1981).
- Thornhill, S.G., and ter Haar, D., "Langmuir Turbulence and Modulational Instability", Phys. Reports 43, 43-99 (1978).

Gardner, C.S., Green, J.M., Kruskal, M.D., and Miura, R.M., "Method for Solving the Korteweg de Vries Equation", Phys. Rev. Lett. 19, 1095-98 (1967).

Ginzburg, V.L., and Zheleznyakov, V.V., Sov. Astron.--AJ, 2, 653 (1958).

Goldman, M.V., "Progress and Problems in the Theory of Type III Solar Radio Emission", Solar Physics 89, 403-442 (1983).

Goldman, M.V., "Strong Turbulence of Plasma Waves", Reviews of Modern Physics 56, 709-735 (1984).

Goldstein, M.L., Smith, R.A., and Papadopoulos, K., "Nonlinear Stability of Solar Type III Radio Bursts. II. Application to Observations near 1 AU", Ap. J. 234, 683-95 (1979).

Grabbe, C.L., "Resource Letter: Plasma Waves and Instabilities", Am. J. Phys. 52, 970-81 (1984).

Gurnett, D.A., Anderson, R.R., "Electron Plasma Oscillations Associated with Type III Radio Bursts", Science 194, 1159-62 (1976).

Gurnett, D.A., Anderson, R.R., and Tokar, R.L., "Plasma Oscillations and the Emissivity of Type III Radio Bursts," pp. 369-79 in M.R. Kundu and T.E. Gergely (eds.), Radio Physics of the Sun, IAU (1980).

Gurnett, D.A., Maggs, J.E., Gallagher, D.L., Kurth, W.S., and Scarf, F.L., "Parametric Interaction and Spatial Collapse of Beam-Driven Langmuir Waves in the Solar Wind," J. Geophys. Res. 86, 8835-41 (1981).

Kadomtsev, B.B., Plasma Turbulence, Academic Press, New York (1965).

Kurth, W.S., Gurnett, D.A., Scarf, F.L., and Poynter, R.L., "Detection of a Radio Emission at 3 kHz in the Outer Heliosphere," Nature 312, 27-31 (1984).

Muldrew, D.B., "An Ionization Duct Explanation of Some Plasma-Line Observations with a 46.8 MHz Radar and with a 430 MHz Radar", J. Geophys. Res. (1985, in press).

Nicholson, D.R., Introduction to Plasma Theory Wiley, New York (1983).

Nicholson, D.R., Goldman, M.V., Hoyng, P., and Weatherall, J.C., "Nonlinear Langmuir Waves during Type III Solar Radio Bursts," Ap. J. 223, 605-19 (1978).

Nicholson, D.R., Payne, G.L., Downie, R.M., and Sheerin, J.P., "Solitons versus Parametric Instabilities during Ionospheric Heating", Phys. Rev. Lett. 52, 2152-55 (1984).

Tsyтович, V.N., Nonlinear Effects in Plasma, Plenum, New York (1968).

Tsyтович, V.N., Theory of Turbulent Plasma, Consultants Bureau, New York (1977).

Utlaut, W.F., and Violette, E.J., "A Summary of Vertical Incidence Radio Observations of Ionospheric Modification", Rad. Sci. 9, 895-903 (1974).

Utlaut, W.F., Violette, E.J., and Paul, A.K., "Some Ionosonde Observations of Ionospheric Modification by Very High Power, High Frequency Ground-Based Transmission", J. Geophys. Res. 75, 6429-35 (1970).

Weatherall, J.C., Sheerin, J.P., Nicholson, D.R., Payne, G.L., Goldman, M.V., and Hansen, P.J., "Solitons and Ionospheric Heating", J. Geophys. Res. 87, 823-832 (1982).

Wild, J.P., Aust. J. Sci. Ser. A3, 541 (1950).

Zakharov, V.E., "Collapse of Langmuir Waves", Soviet Physics JETP 35, 908-914 (1972); Zh. Eksp. Teor. Fiz. 62, 1745-51 (1972).

Zakharov, V.E., and Shabat, A.B., "Exact Theory of Two-Dimensional Self-Focusing and One-Dimensional Self-Modulation of Waves in Nonlinear Media", Sov. Phys. JETP 34, 62-69 (1972); Zh. Eksp. Teor. Fiz. 61, 118-134 (1971).

Zheleznyakov, V.V., and Zaitsev, V.V., "Contribution to the Theory of Type III Solar Radio Bursts. I.", Sov. Astron.--AJ 14, 47-58 (1970).